Comment on "Calculation of ionization balance and electrical conductivity in nonideal aluminum plasma"

Mofreh R. Zaghloul
Center for Energy Research, University of California San Diego, 9500 Gilman Drive, La Jolla, California 92093-0438, USA
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We comment on the use of an inaccurate fitting formula for the energy-average electron-ion momentum transport cross section in the computation of electrical conductivity of nonideal aluminum plasma.

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In a recent paper [1], Kim and Kim introduced and implemented an approach to calculate the ionization balance and electrical conductivity of warm dense aluminum plasma with the Coulomb coupling effect taken into consideration. The evaluation of electrical conductivity in a partially ionized condition is based on a linear mixture rule of additive collision frequencies for electron-ion and electron-neutral interactions. For the electron-ion interaction, the authors adopted an often-used model proposed by Zollweg and Liebermann [2]. The model is governed by two basic modifications to the Spitzer expression [3]: (1) the assumption that $\ln(\Lambda = b_{\mathrm{cut}}/\overline{b_0} \approx \lambda_D/\overline{b_0}) \gg 1$ is not made in the cross-section integral (where b_{cut} is the cutoff impact parameter taken to be the Debye length, λ_D, in Spitzer's theory and $\overline{b_0}$ is an average thermal impact parameter) and thus a more accurate evaluation of that integral was necessary; and (2) the parameter $\Lambda \approx \lambda_D/\overline{b_0}$ is replaced by an effective value Λ_m that prevents the divergence of the transport properties as the parameter $\Lambda \to 0$. The authors of Ref. [2] computed the cross-section integral numerically and proposed the fitting formula $\ln(1+1.4\Lambda^2)^{1/2}$ to replace the ordinary Coulomb logarithm $\ln(\Lambda)$ in the calculation of the effective collision frequency used in the simple computations of plasma transport properties within the framework of the binary-collision cutoff theory. However, in a previous work [4], the cross-section integral (or simply Coulomb logarithm) was derived analytically and computed numerically, showing the inaccuracy of the fitting formula given by Zollweg and Liebermann [2]. The derived exact analytic expression for the cross-section integral (or simply Coulomb logarithm) is expressed in terms of the well-known and tabulated sine and cosine integrals (Si and Ci) and is given as

$$\ln(\Lambda) \to \frac{\pi}{2}\sin\left(\frac{1.5}{\Lambda}\right)\left[1 - \frac{2}{\pi}\left(\mathrm{Si}\left(\frac{1.5}{\Lambda}\right) + \frac{\mathrm{Ci}\left(\frac{1.5}{\Lambda}\right)}{\tan\left(\frac{1.5}{\Lambda}\right)}\right)\right].$$

Values of Si and Ci are also available in many software packages. Comparison between the exact analytic form for the cross-section integral as given above and the fitting formula of Ref. [2] showed a percentage relative error that goes up to higher than 100% as a result of using Zollweg and Liebermann's expression in its proposed range of applicability (see Ref. [4] for a plot of the resulting error).

As Zollweg and Liebermann's expression loses the accuracy for which it was proposed, this expression should not be used or considered any further in the computations of the transport properties of nonideal plasmas.

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